

Constraining the properties of dark energy

Dragan Huterer* and Michael S. Turner†

**Physics Department, University of Chicago, Chicago, IL*

†Physics and Astronomy and Astrophysics Departments, University of Chicago, Chicago, IL

Abstract. The presence of dark energy in the Universe is inferred directly from the accelerated expansion of the Universe, and indirectly, from measurements of cosmic microwave background (CMB) anisotropy. Dark energy contributes about 2/3 of the critical density, is very smoothly distributed, and has large negative pressure. Its nature is very much unknown. Most of its discernible consequences follow from its effect on evolution of the expansion rate of the Universe, which in turn affects the growth of density perturbations and the age of the Universe, and can be probed by the classical kinematic cosmological tests. Absent a compelling theoretical model (or even a class of models), we describe dark energy by an effective equation of state $w = p_X/\rho_X$ which is allowed to vary with time. We describe and compare different approaches for determining $w(t)$, including magnitude-redshift (Hubble) diagram, number counts of galaxies and clusters, and CMB anisotropy, focusing particular attention on the use of a sample of several thousand type Ia supernova with redshifts $z < 1.7$, as might be gathered by the proposed SNAP satellite. Among other things, we derive optimal strategies for constraining cosmological parameters using type Ia supernovae. While in the near term CMB anisotropy will provide the first measurements of w , supernovae and number counts appear to have the most potential to probe dark energy.

INTRODUCTION

Three major lines of evidence point to the existence of a smooth energy component in the universe. Various measurements of the matter density indicate $\Omega_M \simeq 0.3 \pm 0.1$ (e.g., [1]). Recent cosmic microwave background (CMB) results strongly favor a flat (or nearly flat) universe, with the total energy density $\Omega_0 \simeq 1.1 \pm 0.07$ [2]. Finally, there is direct evidence coming from type Ia supernovae (SNe Ia) that the universe is accelerating its expansion, and that it is dominated by a component with strongly negative pressure, $w = p_X/\rho_X < -0.6$ [3, 4]. Two out of three of these arguments would have to prove wrong in order to do away with the smooth component.

Even before the direct evidence from SNe Ia, there was a dark energy candidate: the energy density of the quantum vacuum (or cosmological constant) for which $p = -\rho$. However, the inability of particle theorists to compute the energy of the quantum vacuum – contributions from well understood physics amount to 10^{55} times critical density – casts a dark shadow on the cosmological constant. Another important issue is the coincidence problem: dark energy seems to start dominating the energy budget, and accelerating the expansion of the universe, just around the present time. A number of other candidates have been proposed: rolling scalar field (or quintessence), and network of frustrated topological defects, to name a couple. While these and other models have some motivation and attractive features, none are compelling.

In this work we discuss the cosmological consequences of dark energy that allow its

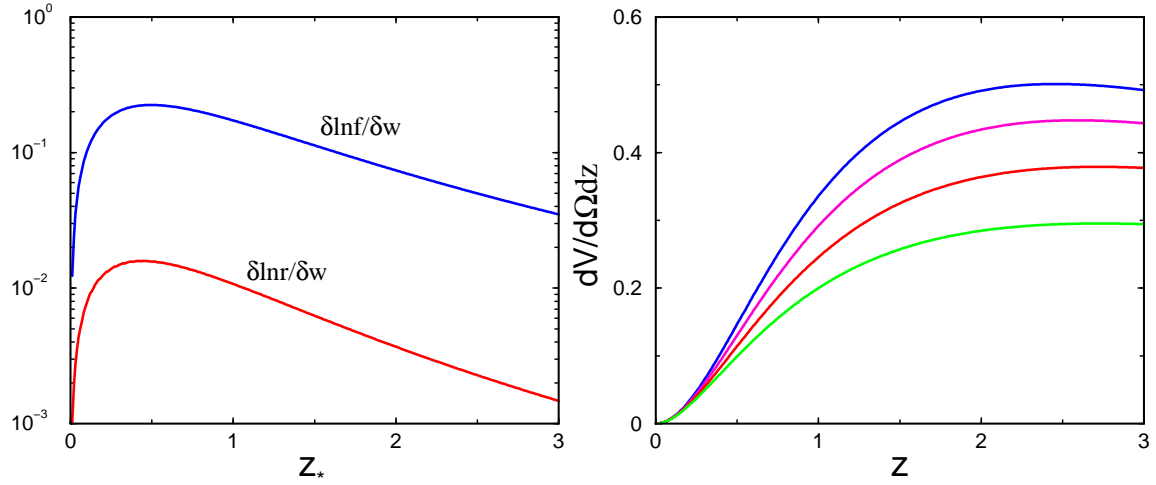


FIGURE 1. Left panel: relative sensitivity of the comoving distance $r(z)$ and the comoving volume element $f(z) \equiv dV/d\Omega dz$ to a localized change in the value of w at redshift z_* . Right panel: comoving volume element $dV/d\Omega dz$ vs. redshift for constant $w = -1, -0.8, -0.6, -0.4$ (from top to bottom).

nature to be probed. We also discuss relative merits of various cosmological probes, focusing particular attention to a large, well-calibrated sample of SNe Ia that would be obtained by the proposed space telescope SNAP¹. We parameterize dark energy by its scaled energy density Ω_X and equation of state w , and assume a fiducial cosmological model with $\Omega_X = 1 - \Omega_M = 0.7$ and $w = -1$, unless indicated otherwise.

COSMOLOGICAL CONSEQUENCES OF THE DARK ENERGY

Dark energy is smooth, and if does clump at all, it does so at very large scales only ($k \sim H_0$). All of its consequences therefore follow from its modification of the expansion rate

$$H(z)^2 = H_0^2 \left[\Omega_M (1+z)^3 + \Omega_X \exp \left[3 \int_0^z (1+w(x)) d \ln(1+x) \right] \right].$$

For a given rate of expansion today H_0 , the expansion rate in the past, $H(z)$, was smaller in the presence of dark energy. Therefore, dark energy increases the age of the universe. The comoving distance $r(z) = \int dz/H(z)$ also increases in the presence of dark energy. The same follows for the comoving volume element $dV/d\Omega dz = r^2(z)/H(z)$.

At small redshifts $r(z)$ is insensitive to w for the simple reason that *all* cosmological models reduce to the Hubble law ($r = H_0^{-1} z$) for $z \ll 1$:

¹ <http://snap.lbl.gov>

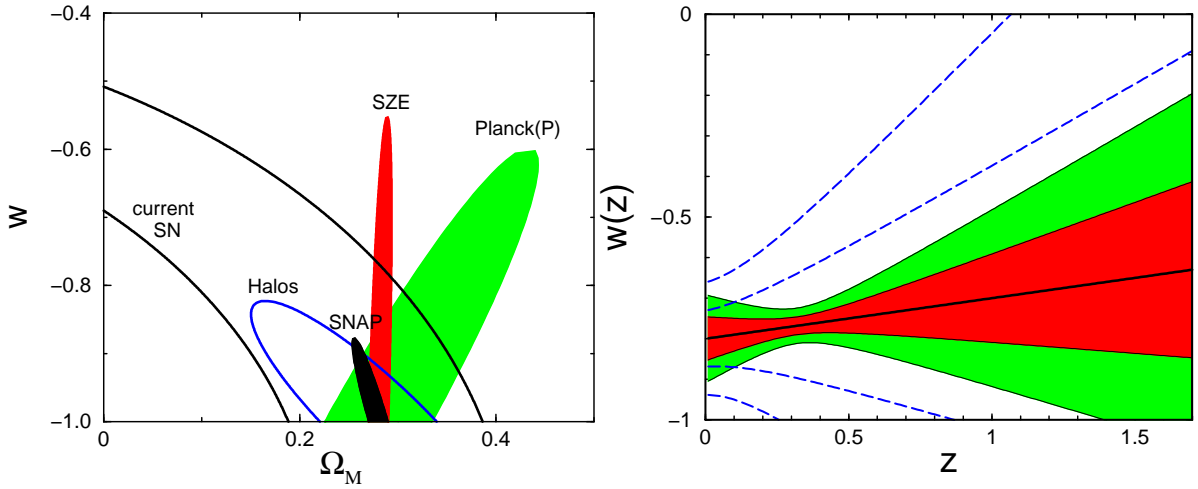


FIGURE 2. Left panel: present and projected future constraints (68% C.L.) on constant w . The Sunyaev-Zeldovich effect (SZE) constraint roughly corresponds to the estimate from [5], while for the halo counts we assumed a random uncertainty of 10% in each of eight redshift bins. Right panel: projected constraints (68% and 95% C.L.) on time-varying w using simulated data expected from SNAP. The fiducial model is $w(z) = -0.8 + 0.1z$ and Ω_M is considered to be known. The broken lines show the effect of assuming a Gaussian uncertainty of 0.05 in Ω_M .

$$r(z) \rightarrow H_0^{-1} \left[z - \frac{3}{4}z^2 - \frac{3}{4}\Omega_X w z^2 + \dots \right] \text{ for } z \ll 1. \quad (1)$$

At redshift greater than about five, the sensitivity of $r(z)$ to a change in w levels off because dark energy becomes an increasingly smaller fraction of the total energy density, $\rho_X/\rho_M \propto (1+z)^{3w}$. Note also that the volume depends upon dark energy parameters through $H(z)$, which contains one integral less than the distance $r(z)$. This is the reason why number-count surveys (which effectively measure $dV/d\Omega dz$) are potentially a strong probe of dark energy.

Constraints on (constant) w

Supernovae are perhaps the strongest probes of dark energy, as the relevant observable $r(z)$ depends only on Ω_M , Ω_X and w ; moreover, SNe probe the optimal redshift range. A powerful supernova program, such as SNAP, with about 2000 SNe distributed at $0.2 < z < 1.7$, would be able to determine w with an accuracy $\sigma_w = 0.05$.

Number-count surveys are potentially also a very strong probe of dark energy. To estimate the expected number density of objects (galaxies, galaxy clusters etc.), one typically uses the Press-Schechter formalism [6]. For example, number counts of galaxy clusters either from an X-ray survey or from Sunyaev-Zeldovich survey can serve as a probe of dark energy [5]. The number of these objects depends upon the growth of structure as well as on cosmological volume, and the former makes the overall constraint complementary to that of SNe Ia. Another example is using halos of fixed

rotational speed [7] to infer the dark energy dependence of the volume element. In order to obtain constraints on dark energy using number-count surveys, control over modeling and systematic errors will be critical.

CMB anisotropy, which mainly probes the epoch of recombination ($z \sim 1000$) is weakly sensitive to the presence of dark energy because dark energy becomes inconsequential at such high redshift. The small dependence comes through the distance to the surface of last scattering which slightly increases in the presence of dark energy, moving the acoustic peaks to smaller scales. However, this effect is small:

$$\frac{\Delta l_1}{l_1} = -0.084\Delta w - 0.23\frac{\Delta\Omega_M h^2}{\Omega_M h^2} + 0.09\frac{\Delta\Omega_B h^2}{\Omega_B h^2} + 0.089\frac{\Delta\Omega_M}{\Omega_M} - 1.25\frac{\Delta\Omega_0}{\Omega_0},$$

which shows that the location of the first peak is least sensitive to w . The left panel of Fig. 2 further illustrates this: even the Planck experiment with polarization information would be able to achieve only $\sigma_w \approx 0.25$ (after marginalization over all other parameters). Nevertheless, CMB constraints are of crucial importance because they complement SNe and other probes, in particular providing the measurement of the total energy density Ω_0 .

The Alcock-Paczynski shape test and the age of the Universe are also sensitive to the presence of dark energy, but they seem somewhat less promising: the former because of the small size of the effect (around 5%); and the latter because the errors in the two needed quantities, H_0 and t_0 , are not likely to become small enough in the near future. Finally, large-scale structure surveys have weak direct dependence upon dark energy simply because this component is smooth on observable scales.

Probing $w(t)$

There is no *a priori* reason to assume constant w , as some models (in particular quintessence) generically produce time-varying w . Here we discuss how to best constrain $w(t)$ (or $w(z)$). We find that $w(z)$ will be much more difficult to constrain than constant w due to additional degeneracies. In order to illustrate prospects for constraining $w(z)$, we use SNAP's projected dataset with 2000 SNe and assume that, by the time this difficult task is seriously attempted, Ω_M and Ω_X will be pinned down accurately by a combination of CMB measurements and large-scale structure surveys.

One of the simplest ways to characterize $w(z)$ is to divide the redshift range into B redshift bins and assume constant equation of state ratio w_i in each. The resulting constraint, however, is poor for $B \geq 3$ (and ideally one would want many more bins). A better way to proceed is to assume linearly varying $w(z)$ expanded around a suitably chosen redshift w_1 [8]

$$w(z) = w_1 + w'(z - z_1).$$

We choose z_1 so as to de-correlate w_1 and w' . The resulting constraint is shown in the right panel of Fig. 2. The constraint is best at $z \approx 0.4$ and deteriorates at lower and higher redshifts. Despite the relatively large uncertainty in the slope ($\sigma_{w'} = 0.16$),

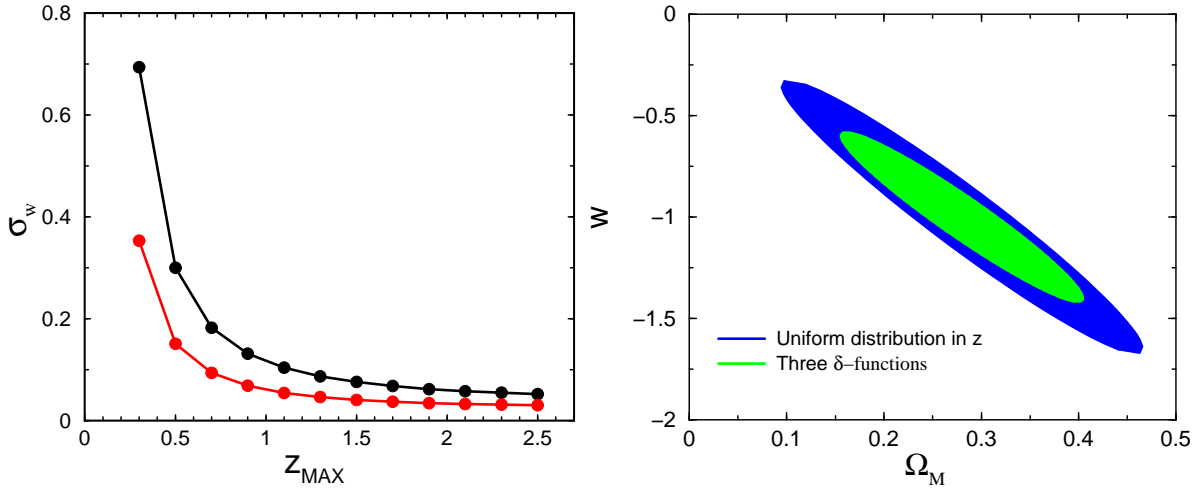


FIGURE 3. Left panel: upper curve shows the constraint upon (constant) w using the (nearly uniform) distribution of 2000 SNe out to z_{max} . Lower curve shows uncertainties using the same number of SNe with mathematically optimal distribution in redshift. Right panel: constraints on Ω_M and w using 100 SNe with uniform (dark) and optimal (light) distribution in redshift.

this analysis may be useful in constraining dark energy models. The parameterization $w(z) = w_1 - \alpha \ln[(1+z)/(1+z_1)]$ yields comparable constraints.

OPTIMAL STRATEGIES

Given the importance and difficulty of probing dark energy, it is worthwhile to consider how to optimize data sets in order to obtain tighter constraints. We address the following question: what is the optimal redshift distribution of SNe Ia in order to best constrain the cosmological parameters?

We choose to minimize the area of uncertainty ellipsoid for P parameters, which (in the Fisher matrix formalism [9]) is equivalent to maximizing $\det F$, where F is the Fisher matrix. Making a few simplifying but reasonable assumptions, in [10] we show that P parameters are always best determined if SNe are distributed as $P + 1$ delta-functions in redshifts, two of which are always at the lowest and highest redshifts available. For example, to best constrain Ω_X and w (assuming a flat universe), SNe should be located at $z = 0$, $z \approx 2/5 z_{\text{max}}$ and $z = z_{\text{max}}$ in equal numbers, where z_{max} is the maximum redshift available. In Fig. 3 we show the merits of optimal distribution. Note, however, that there are other considerations that may favor a more uniform distribution of SNe in redshift – for example, ability to perform checks for dust and evolution.

CONCLUSIONS

We discussed prospects for constraining dark energy. Dark energy is best probed at redshifts roughly between 0.2 and 1.5, where it is dynamically important. Probes of

the low-redshift Universe (supernovae and number counts) seem the most promising, as they only depend upon three cosmological parameters (Ω_M , Ω_X and w), which will be effectively reduced to two (Ω_X and w) when precision CMB measurements determine $\Omega_0 = \Omega_M + \Omega_X$ to better than 1%. CMB is weakly sensitive to the presence of dark energy, but is very important as a complementary probe. Alcock-Paczynski test and the age of the universe seem less promising.

Constraining the equation of state w is the first step in revealing the nature of the dark energy. We show that future surveys will be able to determine constant w quite accurately. A high-quality sample of 2000 supernovae out to redshift $z \sim 1.7$ could determine w to a precision of $\sigma_w = 0.05$. A similar accuracy might be achieved by number counts of galaxies and clusters of galaxies out to $z \sim 1.5$, provided systematics are held in check (e.g., in the case of galaxies, the evolution of the comoving number density needs to be known to better than 5%). Time-varying w will be considerably more difficult to pin down because of additional degeneracies that arise in this case. Nevertheless, interesting constraints may be achieved using high-quality data and assuming a slowly-varying $w(z)$. Consequently, use of complementary measurements and further development of techniques to constrain dark energy will be of crucial importance.

ACKNOWLEDGMENTS

We would like to thank the organizers for the wonderful conference and the “black hole” drink recipe. This work was supported by DOE at Chicago.

REFERENCES

1. Turner, M. S., *Physica Scripta*, **85**, 210 (2000).
2. Jaffe, A. H., et al., *astro-ph/0007333* (2000).
3. Riess, A., et al., *Astron. J.*, **116**, 1009 (1998).
4. Perlmutter, S., et al., *Astrophys. J.*, **517**, 545 (1999).
5. Haiman, Z., Mohr, J. J., and Holder, G. P., *astro-ph/0002336* (2000).
6. Press, W. H., and Schechter, P. L., *Astrophys. J.*, **187**, 425 (1974).
7. Newman, J., and Davis, M., *Astrophys. J. Lett.*, **534**, 11 (2000).
8. Cooray, A. R., and Huterer, D., *Astrophys. J. Lett.*, **513**, 95 (1999).
9. Tegmark, M., Taylor, A. N., and Heavens, A. F., *Astrophys. J.*, **480**, 22 (1997).
10. Huterer, D., and Turner, M. S., *astro-ph/0012510* (2000).